Multiuser Detectors for MIMO DS/CDMA Systems

Fang-Biau Ueng, Shang-Chun Tsai and Jun-Da Chen

EE, NCHU, Taichung, Taiwan. E-mail: fbueng@ee.nchu.edu.tw

We consider a DS/CDMA communication system with K active users. Each user is assigned a unique spreading sequence $\{c_k(t)\}_{k=1}^{K}$ with finite support, $c_k(t) = 0$, $t \notin [0, T_b]$. T_b is the bit duration. The transmitted signal is given by $s(t) = \sum_{k=1}^{K} A_k b_k c_k(t)$, $t \in [0, T_b]$, where A_k is the kth user's amplitude, b_k is the binary data bit transmitted by kth user. $\{b_k\}_{k=1}^{K}$ are assumed to be i.i.d. Bernoulli distributed. Under BPSK modulation, $b_k \in \{-1, +1\}$. The size of each vector is equal to the length of spreading sequence or processing gain L. Then, each vector can be written as $\mathbf{s} = \sum_{k=1}^{K} A_k b_k \mathbf{c}_k = \mathbf{CAb}$, where $\mathbf{s}$ is the transmitter signal vector, the dimension of $\mathbf{s}$ is equal to L at chip interval T_c. $\mathbf{C} \in R^{L\times K}$, $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_K]$. $\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_K$ is the spreading sequence of each user, respectively. We consider a MIMO channel with N transmitted and M received antennas. Assuming that the detector has the knowledge of carrier frequency and phase shift and thus cancelled out in the receiver front end and the fading is slow such that the channel remains constant during one bit time slot of J chip intervals. The received signal at the antenna array can be written as

$$\mathbf{x}(j) = \mathbf{H}\tilde{\mathbf{s}}(j) + \mathbf{n}(j), \ j = 1, \cdots, J \tag{1}$$

where $\mathbf{H}$ is the $M \times N$ MIMO channel matrix coupling the transmitter and receiver antennas. $\mathbf{n}$ is white Gaussian random vector. The one bit received signal can be written as

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} \mathbf{x}(1) & \mathbf{x}(2) & \cdots & \mathbf{x}(J) \end{bmatrix} \\ &= \mathbf{H}\begin{bmatrix} \tilde{\mathbf{s}}(1) & \tilde{\mathbf{s}}(2) & \cdots & \tilde{\mathbf{s}}(J) \end{bmatrix} + \begin{bmatrix} \mathbf{n}(1) & \mathbf{n}(2) & \cdots & \mathbf{n}(J) \end{bmatrix} = \mathbf{HS} + \tilde{\mathbf{N}} \end{aligned} \tag{2}$$

where $\tilde{\mathbf{N}}$ consists of $\mathbf{n}(1), \cdots, \mathbf{n}(J)$ and is also white Gaussian random matrix. The array output signal $\mathbf{r}(j)$ is linearly combined through a complex weight matrix $\mathbf{W}$ to yield the array output signal that can be written as $\mathbf{r}(j) = \mathbf{W}^H\mathbf{x}(j)$, $j = 1, \cdots, J$, where $\mathbf{W}$ is $M \times N$ weight matrix. From (1), we reconstitute the array output signal. Buffering the array output signal into blocks of length J, each block can be written in vector form

$$\tilde{\mathbf{r}} = \tilde{\mathbf{W}}^H \tilde{\mathbf{x}} \tag{3}$$

where $\tilde{\mathbf{W}}$ is $JM \times JN$ weight matrix and $\tilde{\mathbf{x}}$ is $JM \times 1$ received signal vector that can be expressed as

$$\tilde{\mathbf{x}} = \tilde{\mathbf{H}}\mathbf{s} + \tilde{\mathbf{n}} = \tilde{\mathbf{H}}\mathbf{Cb} + \tilde{\mathbf{n}} \tag{4}$$

where $\tilde{\mathbf{H}}$ is $JM \times JN$ MIMO channel matrix.

TLS Detector

The robust detector is based on the TLS algorithm[3] and we name it as the "TLS detector[4]". As signal propagates through the MIMO channel, it is usually subject to multipath fading such that it arrives at the destination along a number of different paths. This leads to the signature waveform distortion[5]. Combine (3) and (4), the antenna array output signal can be written as

$$\tilde{\mathbf{r}} = \tilde{\mathbf{W}}^H \tilde{\mathbf{H}}\mathbf{Cb} + \tilde{\mathbf{W}}^H \tilde{\mathbf{n}} \tag{5}$$

0-7803-9068-7/05/$20.00 ©2005 IEEE

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **01 JAN 2005**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**

4. TITLE AND SUBTITLE **Multiuser Detectors for MIMO DS/CDMA Systems**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **EE, NCHU, Taichung, Taiwan**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM001846, Applied Computational Electromagnetics Society 2005 Journal, Newsletter, and Conference.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**UU**	**4**	

The summary of the TLS-based algorithm can be described as follows, the optimum $\mathbf{E}_s$ is composed of the K largest (nonzero) eigenvectors of the matrix $\begin{bmatrix} \mathbf{C} & \tilde{\mathbf{r}} \end{bmatrix} \begin{bmatrix} \mathbf{C} & \tilde{\mathbf{r}} \end{bmatrix}^H$:

1.Perform eigendecomposition on $\begin{bmatrix} \mathbf{C} & \tilde{\mathbf{r}} \end{bmatrix} \begin{bmatrix} \mathbf{C} & \tilde{\mathbf{r}} \end{bmatrix}^H$ to obtain $\mathbf{E}_s$
2.Find projection operator $\mathbf{P}_c = \mathbf{E}_s\mathbf{E}_s^H$
3.The TLS detector $\hat{\mathbf{b}}_{TLS} = \left(\mathbf{C}^T\mathbf{P}_c\mathbf{C}\right)^{-1}\mathbf{C}^T\mathbf{P}_c\tilde{\mathbf{r}}/L = \mathbf{T}_{TLS}^T\tilde{\mathbf{r}}/L$
4.The decision rule $\hat{\mathbf{b}}_{TLS} = sgn\left(\mathbf{T}_{TLS}^T\tilde{\mathbf{r}}/L\right)$

ESPRIT Detector

The summary of the ESPRIT-based algorithm can be described as follows:

1.Obtain an estimate $\hat{\mathbf{R}}_{xx}$ of $\mathbf{R}_{xx}$ from the measurements $\mathbf{x}$
2.Perform eigendecomposition on $\hat{\mathbf{R}}_{xx}$ to obtain $\hat{\mathbf{U}}_s$
3.Solve $\hat{\mathbf{U}}_{s1} = \hat{\mathbf{\Psi}}\hat{\mathbf{U}}_{s2}$ for $\hat{\mathbf{\Psi}}$
4.LS solution $\hat{\mathbf{\Psi}}_{LS} = \left(\hat{\mathbf{U}}_{s1}^H\hat{\mathbf{U}}_{s1}\right)^{-1}\hat{\mathbf{U}}_{s1}^H\hat{\mathbf{U}}_{s2}$
TLS solution $\hat{\mathbf{\Psi}}_{TLS} = -\mathbf{U}_{12}\mathbf{U}_{22}^{-1}$
$\mathbf{R}_u = \begin{bmatrix} \hat{\mathbf{U}}_{s1}^H \\ \hat{\mathbf{U}}_{s2}^H \end{bmatrix} \begin{bmatrix} \hat{\mathbf{U}}_{s1} & \hat{\mathbf{U}}_{s2} \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} \\ \mathbf{U}_{21} & \mathbf{U}_{22} \end{bmatrix} \Lambda_u \begin{bmatrix} \mathbf{U}_{11}^H & \mathbf{U}_{12}^H \\ \mathbf{U}_{21}^H & \mathbf{U}_{22}^H \end{bmatrix}$
5.Obtain eigenvalue $\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_N$ of $\hat{\mathbf{\Psi}}_{LS}$ and $\hat{\mathbf{\Psi}}_{TLS}$
6.Find $\hat{\boldsymbol{\theta}}_r = \sin^{-1}\left(\frac{\ln\hat{\lambda}_n}{jk_c d}\right)$, $n = 1, \cdots N$
7.Obtain channel estimates $\hat{\mathbf{H}}$ and $\breve{\mathbf{H}}$
8.The ESPRIT detector $\hat{\mathbf{b}}_{ESPRIT} = \left(\tilde{\mathbf{W}}^H\breve{\mathbf{H}}\mathbf{C}\right)^+\tilde{\mathbf{r}}/L = \mathbf{T}_{ESPRIT}^T\tilde{\mathbf{r}}/L$
9.The decision rule $\hat{\mathbf{b}}_{ESPRIT} = sgn\left(\mathbf{T}_{ESPRIT}^T\tilde{\mathbf{r}}/L\right)$

LMS Receiver

We propose an adaptive receiver that is based on the LMS algorithm and we name it as the "LMS receiver". The despreader output $\hat{\mathbf{b}}(n)$ can be written as $\hat{\mathbf{b}}(n) = \mathbf{C}^H\tilde{\mathbf{r}}(n)/L = \mathbf{C}^T\tilde{\mathbf{W}}^H(n)\tilde{\mathbf{x}}(n)/L$. A linear receiver that comprising a weight matrix $\tilde{\mathbf{W}}$ operates on the input vector $\tilde{\mathbf{x}}$ to yield the output $\hat{\mathbf{b}}$. The weight matrix $\tilde{\mathbf{W}}$ is chosen to minimize the cost function

$$J_{LMS} = E\left[\left\|\mathbf{C}^H\tilde{\mathbf{W}}^H(n)\tilde{\mathbf{x}}(n)/L - \mathbf{b}(n)\right\|^2\right] = E\left[\|\mathbf{e}(n)\|^2\right] \tag{6}$$

The adaptive algorithm is as $\tilde{\mathbf{W}}(n+1) = \tilde{\mathbf{W}}(n) - 2\mu\tilde{\mathbf{x}}(n)\mathbf{e}^H(n)\mathbf{C}^H/L$. In order to improve the LMS-type receiver, we propose an advanced scheme named as the "LMS sequence revising (LMS-SR) receiver". The new structure uses LMS algorithm to update despreading sequence. Taking gradient with respect to the despreading sequence, we get $\frac{\partial J_{LMS}}{\partial \mathbf{C}} = 2\tilde{\mathbf{W}}^H(n)\tilde{\mathbf{x}}(n)\mathbf{e}^H(n)/L$. The sequence update equation is $\mathbf{C}(n+1) = \mathbf{C}(n) - \mu\frac{\partial J_{LMS}}{\partial \mathbf{C}}$.

ESPRIT Based LMS Receiver

The summary of the ESPRIT-LMS algorithm can be described as follows:

1.Obtain an estimate $\hat{\mathbf{R}}_{xx}$ of $\mathbf{R}_{xx}$ from the measurements $\mathbf{x}$
2.Perform eigendecomposition on $\hat{\mathbf{R}}_{xx}$ to obtain $\hat{\mathbf{U}}_s$
3.Solve $\hat{\mathbf{U}}_{s1} = \hat{\mathbf{\Psi}}\hat{\mathbf{U}}_{s2}$ for $\hat{\mathbf{\Psi}}$
4.Obtain eigenvalues $\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_N$ of $\hat{\mathbf{\Psi}}_{LS}$ and $\hat{\mathbf{\Psi}}_{TLS}$
5.Find $\hat{\boldsymbol{\theta}}_r = \sin^{-1}\left(\frac{\ln \hat{\lambda}_n}{jk_c d}\right)$, $n = 1, \cdots N$
6.Obtain channel estimates $\hat{\mathbf{H}}$ and $\breve{\mathbf{H}}$
7.Initialization $\tilde{\mathbf{W}}(0) = \left(\breve{\mathbf{H}}^{+}\right)^H$
8.The error signal $\mathbf{e} = \hat{\mathbf{b}} - \mathbf{b}$
9.Weight update $\tilde{\mathbf{W}}(n+1) = \tilde{\mathbf{W}}(n) - 2\mu\tilde{\mathbf{x}}\mathbf{e}^H\mathbf{C}^H / L$
10.Spreading sequence update $\mathbf{C}(n+1) = \mathbf{C}(n) - 2\mu\tilde{\mathbf{W}}^H\tilde{\mathbf{x}}\mathbf{e}^H / L$

Simulation Results

We compare the performance between the six proposed receivers (TLS-based detector, ESPRIT-based detector and their adaptive detectors) under the following MIMO DS/CDMA environments:

	Transmit ant. size	Receive ant size	User number	multipath number
case '1'	$N = 4$	$M = 16$	$K = 4$	$P = 5$
case '2'	$N = 4$	$M = 16$	$K = 32$	$P = 20$

References

[1] A. M. Sayeed, "Deconstructing Multi-Antenna Fading Channels," IEEE Trans. on Signal Processing (special issue on MIMO systems), pp. 2563-2579, Oct. 2002.

[2] H. Artes, D. Seethaler and F. Hlawatsch, "Efficient detection algorithms for MIMO channels: a geometrical approach to approximate ML detection," Signal Processing, IEEE Transactions on [see also Acoustics, Speech, and Signal Processing, IEEE Transactions on], vol. 51, Issue. 11, pp. 2808-2820, Nov. 2003.

[3] G.H. Golub and C.F. Van Loan, *Matrix Computations*, 2nd ed., Johns Hopkins university Press 1989.

[4] Wei-Chiang Wu and Shang-Chun Tsai, "A TLS-based Linear Multiuser detector for Synchronous DS/CDMA Communication System," COM-1-3, Proceeding of the 2002 National Symposium on Telecommunications.

[5] H. Liu and G. Xu, "A subspace method for signature waveform estimation in synchronous CDMA systems," IEEE Trans. Commun., vol. COM-44, No.10, pp. 1346-1354, Oct. 1996.

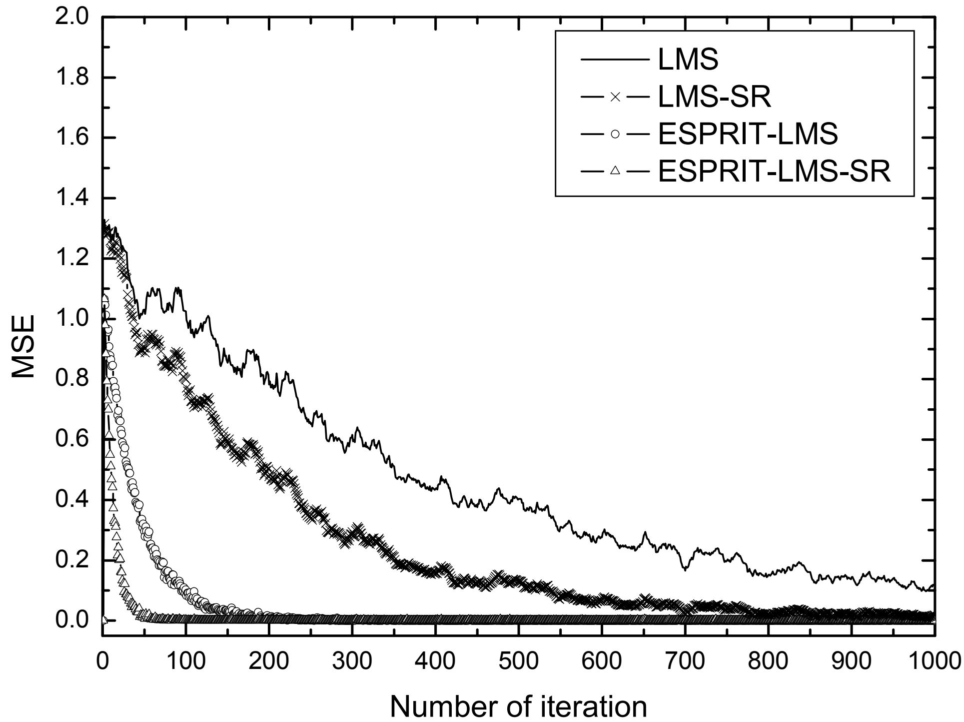

Figure 1: Learning curve comparison between the four proposed algorithm in case '1'

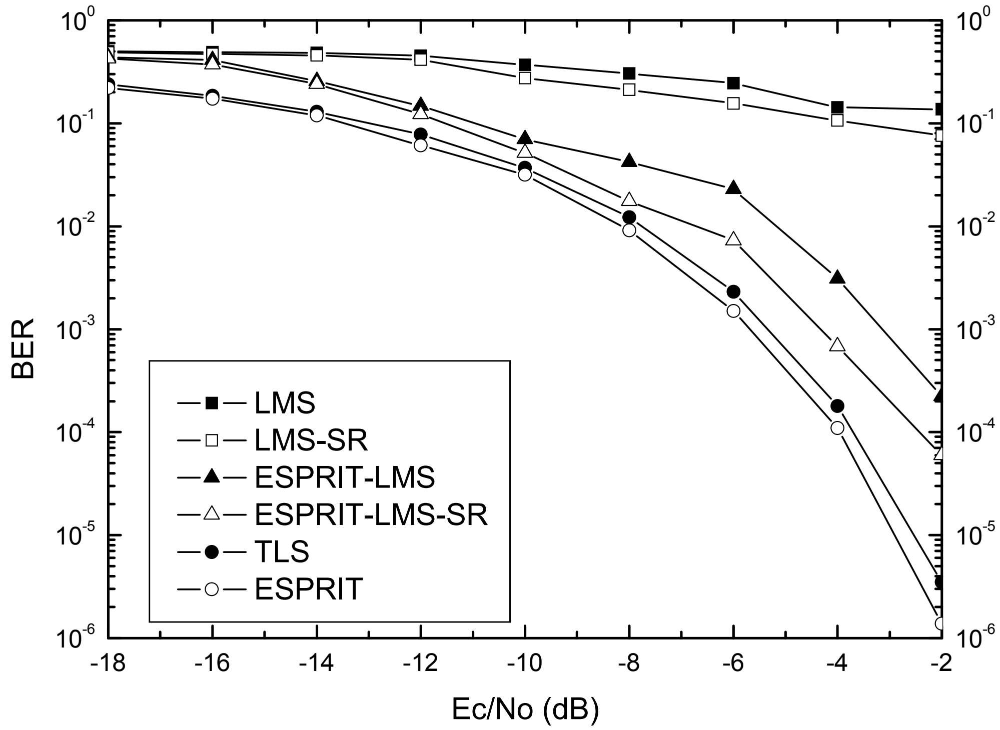

Figure 2: Bit error rate performance comparison between the proposed receivers in case '2'